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Statistical data analysis method for multi-zonal airflow measurement using multiple kinds of perfluorocarbon tracer gas

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ABSTRACT

Conventional multi-zonal ventilation measurement methods by multiple types of perfluorocarbon tracers use a number of different gases equal to the number of zones (n). The possible $n \times n$+n airflows are estimated from the mass balance of the gases and the airflow balance. However, some airflows may not occur because of inter-zonal geometry, and the introduction of unnecessary, unknown parameters can impair the accuracy of the estimation. Also, various error factors often yield an irrational negative airflow rate. Conventional methods are insufficient for the evaluation of error. This study describes a way of using the least-squares technique to improve the precision of estimation and to evaluate reliability. From the equations' residual, the error variance–covariance matrix Λ_q of the estimated airflow rate error is deduced. In addition, the coefficient of determinant using the residual sum of squares and total variation is introduced. Furthermore, the error matrix ${}_m\Lambda_q$ from the measurement errors in the gas concentration and gas emission rate is deduced. The discrepancy ratio of the model premises is defined by dividing the diagonal elements of the former by those of the latter. Moreover, the index of irrationality of the estimated negative airflow rate is defined, based on the different results of the three estimation methods. Some numerical experiments are also carried out to verify the flow rate estimation and the reliability evaluation theory.

1. Introduction

A single-zone model is in practical use in ventilation measurements that use tracer gas. However, an actual building consists of multiple zones, and it is problematic to assume the uniformity of tracer gas diffusion inside a building. The constant-concentration method utilizes a device that controls the gas injection rate to multiple zones and maintains a uniform concentration, but this method cannot measure inter-zonal flow rates, and only the directly invading outdoor airflow rates are estimated. Moreover, stairwells and corridors are often important paths for building ventilation, making the measurement of these flow rates essential. Therefore, many researchers are working to develop multi-zonal ventilation measurement methods to allow the estimation of inter-zonal airflows. Some comprehensive explanations of theoretical and practical aspects of the ventilation measurement are described in several publications [1,2].

Multi-zonal measurement methods currently in the research stage are broadly divided into two categories, one that uses a single tracer gas and the other that uses multiple tracer gases.

The first author of the present paper devised a thermal network model that is a generic framework of spatially discretized heat transfer models. This model is formulated with a simultaneous ordinary differential equation system known as the state space equation in modern control theory. This is a model for a general diffusion system, and also represents single gas diffusion in a multi-zonal system. A system parameter identification theory was deduced from this model using the least-squares method. This theory allowed the derivation of a sequential identification method implemented at every sampling time step, a batch method that produces a result at selected longer time intervals, and an error evaluation method that uses the equation residual and measurement error [3].

The first-generation measurement system based on this system parameter identification theory was developed using three desktop and laptop computers, a specially constructed multiple-point gas injection and air sampling device, and a gas concentration analyzer, which made the system bulky and impractical. Scheduled intermittent gas injection is used in this system. Next, an improved second-generation measurement system was developed using

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Nomenclature

- m_i effective mixing volume of zone i (m^3)
- $q_{i,j}$ airflow rate from zone j to i (m^3/h)
- c_i tracer gas concentration (kg/m^3)
- g_i tracer gas generation rate (kg/h)
- n number of zones inside the building (n+1 indicates the outdoor zone)
- T measurement term in which tracer gas emission and absorption taking place (h)
- $\tilde{m}_i$ concentration derivative-weighted term average effective mixing volume (m^3)
- $\tilde{q}_{i,j}$ concentration-weighted term average air flow rate (m^3/h)
- $\bar{c}_i$ term average concentration (kg/m^3)
- $\tilde{g}_i$ term average gas generation rate (kg/h)
- subscript p measurement term number
- np total number of measurement terms
- subscript k tracer gas kind number
- nk total number of types of tracer gas
- subscript l sampling point number inside a zone
- nl total number of points inside a zone, equal in every zone
- ${}_{p,k,l}\mathbf{c}$ vector of term average concentrations depending on conditions p, k, l size(n+1)
- ${}_{p,k}\mathbf{g}$ vector of term average tracer gas generation depending on conditions p, k size(n)
- $\mathbf{Q}$ flow rates matrix defined by Eqs. (11) and (14) size($n \times (n$+1))
- $\mathbf{q}$ vector of concentration-weighted term average airflow rates q_{ij} size(nq)
- nq total number of airflow rates to be estimated
- ${}_{p,k,l}\mathbf{Z}$ matrix of concentrations defined by Eqs. (16) and (17) size($n \times nq$)
- $\mathbf{C}$ flow rates continuity constraints matrix defined by Eq. (19) size($n \times nq$)
- $\hat{\mathbf{q}}$ vector of estimated flow rates size(nq)
- ${}_{p,k,l}\mathbf{e}$ estimated gas and air flow balance combined equation error size(2n)
- ${}_{p,k,l}\mathbf{F}$ gas and air flow balance combined matrix for flow rates size(2$n \times nq$)
- ${}_{p,k}\mathbf{d}$ gas and air flow balance combined vector for flow rates size(2n)
- J least squares evaluation function (scalar)
- ${}_{p,k,l}\mathbf{v}$ residual vector of gas and air flow balance combined equation size(2n)
- $E(\,)$ statistical operator for the expecting variables or matrix in parentheses
- nt total number for various conditions by p, k, and l, $nt = np \times nk \times nl$
- $\mathbf{\Lambda}_q$ variance–covariance matrix by error propagation from the equation residual expected matrix size($nq \times nq$)
- $s(\hat{\mathbf{q}})$ residual sum of squares for estimated flow rates (scalar)
- ${}_{p,k,l}\mathbf{y}$ vector for total variation calculation size(2n)
- $\bar{\mathbf{y}}$ term average vector of total variation size(2n)
- s_y total variation by tracer gas generation (scalar)
- COD coefficient of determinant
- ${}_{p,k}\mathbf{c}$ vector of term average concentrations depending on conditions p, k size(n+1)
- ${}_{p,k}\mathbf{s}_c$ vector of concentration measurement error depending on conditions p, k size(n+1)
- ${}_{p,k}\bar{\bar{\mathbf{c}}}$ vector of term-average concentrations with no measurement error depending on conditions p, k size(n+1)
- ${}_{p,k}\boldsymbol{\sigma}_c$ vector of concentration measurement error standard deviation depending on conditions p, k size(n+1)
- ${}_{p,k}\mathbf{g}$ vector of term-average tracer gas generation rate depending on conditions p, k size(n)
- ${}_{p,k}\mathbf{s}_g$ vector of gas generation rate measurement error depending on conditions p, k size(n)
- ${}_{p,k}\bar{\bar{\mathbf{g}}}$ vector of term-average tracer gas generation rate with no measurement error depending on conditions p, k size(n)
- ${}_{p,k}\boldsymbol{\sigma}_g$ vector of gas generation rate measurement error standard deviation depending on conditions p, k size(n)
- ${}_{p,k}\boldsymbol{\varepsilon}$ vector of gas and airflow balance combined equation error caused only by measurement error depending on conditions p, k size(2n)
- ${}_{p,k}\boldsymbol{\varepsilon}_u$ vector of tracer gas mass balance equation error caused only from measurement error depending on conditions p, k size(n)
- ${}_m\mathbf{\Lambda}_q$ variance–covariance matrix solely from measurement error size($nq \times nq$)
- β_j index of discrepancy from model premises for jth flow rate in vector $\mathbf{q}$
- $\sigma^2_{\lambda ij}$ jth diagonal element of variance–covariance matrix by error propagation from the equation residual expected matrix
- ${}_m\sigma^2_{\lambda ij}$ jth diagonal element of variance–covariance matrix by error propagation from the measurement error expected matrix
- $\bar{\beta}$ average index of discrepancy from model premises for all flow rates
- $\bar{q}_{i,j}$ average of flow rate for three estimation methods, normal least squares, modified results of normal least squares and non-negative least squares
- ${}_mq_{i,j}$ estimated flow rate from zone j to i by mth estimation method
- ${}_3q_{i,j}$ estimated flow rate from zone j to i by non-negative least squares method
- σ^2_{qij} variance of flow rate for three estimation methods
- $\bar{\sigma}^2_q$ average of variance of flow rate for three estimation methods
- $\bar{q}$ average of all flow rates estimated by non-negative least squares
- R_a irrationality index of estimated negative flow rate $R_a = \bar{\sigma}_q/\bar{q}$

compact commercially available instruments and a non-negative least-squares for data analysis was also introduced [4]. However, the current measurement system requires inconvenient tubing preparation and has interpolation error problems since all zones cannot be measured simultaneously, factors that impair its practicality and accuracy. These problems should be solved by the third-generation system, which will be a dispersed unit placement system with compact units containing a gas emitter and a concentration monitor placed in each zone.

The principle of least squares has been applied in several forms, based on published studies: recursive least squares [5], quadratic programming [6] and nonlinear least squares [7]. Also, the actual measurement error for multicelled buildings using a single gas has been investigated [8].

The multiple tracer gas method allows the use of a rather simple mathematical procedure to estimate inter-zonal airflows, because it is easy to obtain a larger number of equations than with a single gas. Furthermore, the required instruments are simpler.

A multiple perfluorocarbon tracer (PFT) method using the steady-state assumption has been developed [9], and also a proposal for estimation error analysis based on measurement error. A comprehensive theory has been proposed that improves estimation accuracy and error evaluation using a covariance matrix incorporating measured data and prior information [10], and recently a new multi-tracer measurement method has been reported [11]. The different tracer gas release, air sampling and analysis method were devised [12].

In Japan, multiple PFT method field tests have been reported using 3 gases [13] and 4 gases [14] for multi-zonal airflow measurement.

Generally, in the case of n zones there may be $n \times (n+1)$ airflows to be estimated. Steady-state gas mass balance equations are assumed for each zone. When n types of different PFTs are diffused in each zone, n^2 balance equations are obtained and n airflow balance equations are also established. The conventional procedure that obtains and solves equations totaling $n \times (n+1)$, equal to the maximum number of unknown quantities, is the basis of the deterministic method.

The conventional method, however, provides a reliability evaluation method that cannot adequately determine the extent to which the measurement premises are met. These premises include the uniformity of concentration inside the zone, the assumption of a steady state, and the invariability or linearity of the equation parameters. Conventional error evaluation only takes measurement errors into consideration, and errors due to unsatisfied premises have not been taken into account despite their significance. In addition, conventional methods require the estimation of unnecessary airflow that cannot exist because of the geometrical relation between zones. Therefore, irrational negative flow rates are often obtained with conventional methods.

In order to resolve these problems, we should regard them as an optimization problem to minimize the equation error with the airflow rates to be estimated. Also in this optimization non-negative constraints for these airflow rates should be considered. One of the optimizing methods is the least squares which is a statistical method utilizing more equations than the number of airflows from measurements and which also realizes estimation reliability evaluation. In this paper, newly devised error evaluation indices are described with similar models and solutions as reported in a previous paper [3].

2. Basic mathematical model

When there are n zones indoors, a mass balance equation for some type of tracer gas in the ith zone is described as follows. The open air is considered to be a zone, with its zone number defined as $n+1$. Airflow from zone j to i is defined and denoted q_{ij}. The gas concentration in zone i is defined and denoted c_i, the rate of gas generated as g_i, and effective mixing volume as m_i:

$$m_i\dot{c}_i = \sum_{j=1}^{n+1} q_{i,j}c_j - \sum_{j=1}^{n+1} q_{j,i}c_i + g_i \quad (1)$$

By integrating this by the term T, this equation can be written as

$$\frac{1}{T}\int_0^T m_i\dot{c}_i\,dt = \frac{1}{T}\sum_{j=1}^{n+1}\int_0^T q_{i,j}c_j\,dt - \frac{1}{T}\sum_{j=1}^{n+1}\int_0^T q_{j,i}c_i\,dt + \frac{1}{T}\int_0^T g_i\,dt \quad (2)$$

where the concentration-weighted term average parameters are defined as follows. For the effective mixing volume m, the derivative of concentration, and for the flow rate q, the concentration itself is adopted for a weighting coefficient. These concentration-weighted term-average parameters are denoted by a superscript tilde:

$$\tilde{m}_i = \frac{\int_0^T m_i\dot{c}_i\,dt}{\int_0^T \dot{c}_i\,dt} \quad (3)$$

$$\tilde{q}_{i,j} = \frac{\int_0^T q_{i,j}c_j\,dt}{\int_0^T c_j\,dt} \quad (4)$$

$$\tilde{q}_{j,i} = \frac{\int_0^T q_{j,i}c_i\,dt}{\int_0^T c_i\,dt} \quad (5)$$

The concentration-weighted term average flow rate $q_{i,j}$ incoming to zone i from outdoor can be evaluated by coupling with the flow balance equation described later. The gas concentration time varying measurement data are not recorded in PFT method. These equations of concentration weighted term average flow rates cannot actually be calculated but also there is no need for these actual calculations. These equations are described to consider the tendency that the estimated flow rates are smaller than the simple term average flow rates.

Using these parameter definitions, the next Eq. (6) is written as

$$\tilde{m}_i\frac{1}{T}[c_i(T) - c_i(0)] = \sum_{j=1}^{n+1}\tilde{q}_{i,j}\frac{1}{T}\int_0^T c_j\,dt - \sum_{j=1}^{n+1}\tilde{q}_{j,i}\frac{1}{T}\int_0^T c_i\,dt + \frac{1}{T}\int_0^T g_i\,dt \quad (6)$$

By securing enough terms T, the left side can be approximated to 0. The result of the time-integral of concentration on the right side divided by T is the average term concentration. The rate of gas generation becomes the average term generation rate. These are expressed with a superscript bar in the similar way:

$$\bar{c}_j = \frac{1}{T}\int_0^T c_j\,dt \quad (7)$$

$$\bar{g}_i = \frac{1}{T}\int_0^T g_i\,dt \quad (8)$$

As a result, following approximated Eq. (9) can be obtained.

This will be the basic equation model used in the least squares estimation. However, the airflow rates to be estimated are not the term-averaged values but the concentration-weighted term-average values. This type of multi-zonal gas dispersion system is such that if the flow rates are large the concentration will be low, and conversely if the flow rates are small the concentrations will be high. From this, the estimated flow rates are slightly smaller than the simple term-average flow rates. This predicted tendency will be verified in this paper's numerical example:

$$\sum_{j=1}^{n+1}\tilde{q}_{i,j}\bar{c}_j - \sum_{j=1}^{n+1}\tilde{q}_{j,i}\bar{c}_i + \bar{g}_i = 0 \quad (9)$$

As this theory is based on a statistical method, measured values should be obtained under as many conditions as possible. The number of gas types, k, should be greater than the number of zones, nk. The measurement term, p, should be greater than 1, np. The number of points, l, in the zone at which to measure the concentration should be greater than 1, nl. In this way, the estimation accuracy can be improved and errors properly evaluated. Eq. (10) is rewritten as follows using conditions p, k, and l:

$$\sum_{j=1}^{n+1} \tilde{q}_{i,j} \cdot {}_{p,k,l}\bar{c}_j - \sum_{j=1}^{n+1} \tilde{q}_{j,i} \cdot {}_{p,k,l}\bar{c}_i + {}_{p,k}\bar{g}_i = 0 \tag{10}$$

This can be expressed as a matrix with Eq. (11) by defining the concentration and gas generation rate vectors respectively with Eqs (12) and (13). Here, t on the left shoulder means the transpose of the matrix:

$$\begin{bmatrix} -\sum_{j=1}^{n+1}\tilde{q}_{j,1} & \tilde{q}_{1,2} & \cdots & \tilde{q}_{1,n} & \tilde{q}_{1,n+1} \\ \tilde{q}_{2,1} & -\sum_{j=1}^{n+1}\tilde{q}_{j,2} & \cdots & \tilde{q}_{2,n} & \tilde{q}_{2,n+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{q}_{n,1} & \tilde{q}_{n,2} & \cdots & -\sum_{j=1}^{n+1}\tilde{q}_{j,n} & \tilde{q}_{n,n+1} \end{bmatrix} \begin{bmatrix} {}_{p,k,l}\bar{c}_1 \\ {}_{p,k,l}\bar{c}_2 \\ \vdots \\ {}_{p,k,l}\bar{c}_n \\ {}_{p,k,l}\bar{c}_{n+1} \end{bmatrix} + \begin{bmatrix} {}_{p,k}\bar{g}_1 \\ {}_{p,k}\bar{g}_2 \\ \vdots \\ {}_{p,k}\bar{g}_n \end{bmatrix} = \mathbf{0} \tag{11}$$

$${}_{p,k,l}\mathbf{c} = {}^{t}({}_{p,k,l}\bar{c}_1, \ldots, {}_{p,k,l}\bar{c}_n, {}_{p,k,l}\bar{c}_{n+1}) = \begin{pmatrix} {}_{p,k,l}\bar{c}_1 \\ \vdots \\ {}_{p,k,l}\bar{c}_n \\ {}_{p,k,l}\bar{c}_{n+1} \end{pmatrix} \tag{12}$$

$${}_{p,k}\mathbf{g} = {}^{t}({}_{p,k}\bar{g}_1, \ldots, {}_{p,k}\bar{g}_n) \tag{13}$$

$$\mathbf{Q} \cdot {}_{p,k,l}\mathbf{c} + {}_{p,k}\mathbf{g} = 0 \tag{14}$$

Concentration in the open air is generally represented as nonzero. Matrix $\mathbf{Q}$ is $n \times (n+1)$ including the airflow. The rightmost column contains the airflow entering from the open air into each zone.

3. Regression equation and solution of least squares method

In order to obtain solutions for multiple airflows, vector $\mathbf{q}$ including them is defined as shown in Eq. (15). Elements in the vector can be freely ordered. Eq. (14) is changed to an equation for the airflow $\mathbf{q}$. Elements of matrix $\mathbf{Z}$ of $n \times nq$ for vector $\mathbf{q}$ of size nq are determined as follows. Here nq is the number of airflows of unknown size. When the mth element in vector $\mathbf{q}$ is q_{ij}, the ith row mth column element or (i, m) element in the matrix is c_j and (j, m) element is $-c_j$. If i or j exceeds n, they are excluded

$$\mathbf{q} = {}^{t}(\ldots, \tilde{q}_{i,j}, \ldots) \tag{15}$$

$$\begin{bmatrix} \cdots & m\text{th column} & \cdots & \cdots \\ \ldots i\text{th } row \ldots & {}_{p,k,l}\bar{c}_j & \cdots & \cdots \\ \ldots j\text{th row} \ldots & -{}_{p,k,l}\bar{c}_j & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ \tilde{q}_{i,j}(m\text{th row}) \\ \vdots \\ \vdots \end{bmatrix} + \begin{bmatrix} {}_{p,k}\bar{g}_1 \\ {}_{p,k}\bar{g}_2 \\ \vdots \\ {}_{p,k}\bar{g}_n \end{bmatrix} = \mathbf{0} \tag{16}$$

$${}_{p,k,l}\mathbf{Z} \cdot \mathbf{q} + {}_{p,k}\mathbf{g} = \mathbf{0} \tag{17}$$

Constraint of airflow balance should be also considered. For instance, the airflow balance equation in the ith zone is expressed by Eq. (18). This is described for all zones and rewritten as follows with vector $\mathbf{q}$ of the airflow. Elements of matrix $\mathbf{C}$ of $n \times nq$ for $\mathbf{q}$ are determined as follows. When the mth element in $\mathbf{q}$ is q_{ij}, the element (i, m) in this matrix is 1 and the element (j, m) is -1. They are excluded if i or j exceeds n:

$$\sum_{j=1}^{n+1} \tilde{q}_{i,j} - \sum_{j=1}^{n+1} \tilde{q}_{j,i} = 0 \tag{18}$$

$$\begin{bmatrix} \cdots & m\text{th column} & \cdots & \cdots \\ \ldots i\text{th row} \ldots & 1 & \cdots & \cdots \\ \ldots j\text{th row} \ldots & -1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ \tilde{q}_{i,j}(m\text{th row}) \\ \vdots \\ \vdots \end{bmatrix} = \mathbf{0} \tag{19}$$

$$\mathbf{C} \cdot \mathbf{q} = \mathbf{0} \tag{20}$$

Error $\mathbf{e}$ in the equation for the least squares method is defined in the following equation with $\mathbf{Z}$, $\mathbf{g}$ and $\mathbf{C}$ as described above. Combined matrix $\mathbf{F}$ and vector $\mathbf{d}$ are defined as shown in the following equation to simplify the description.

In an earlier theory [3], this type of continuity or mass conservation constraints were used to eliminate the unknown parameters. However, when applying non-negative least squares algorithms to these reduced parameters, the hidden parameters are not subjected to a non-negative condition. Therefore, the present theory incorporates these constraints into the minimizing function and not into the model itself:

$${}_{p,k,l}\mathbf{e} = \begin{bmatrix} {}_{p,k,l}\mathbf{Z} \\ \mathbf{C} \end{bmatrix} \cdot \hat{\mathbf{q}} + \begin{bmatrix} {}_{p,k}\mathbf{g} \\ \mathbf{0} \end{bmatrix} = {}_{p,k,l}\mathbf{F} \cdot \hat{\mathbf{q}} + {}_{p,k}\mathbf{d} \tag{21}$$

The evaluation function J of this error $\mathbf{e}$ is defined with the following equation:

$$\begin{aligned} J &= \sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}_{p,k,l}\mathbf{e} \cdot {}_{p,k,l}\mathbf{e} \\ &= \sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}({}_{p,k,l}\mathbf{F} \cdot \mathbf{q} + {}_{p,k}\mathbf{d})({}_{p,k,l}\mathbf{F} \cdot \mathbf{q} + {}_{p,k}\mathbf{d}) \end{aligned} \tag{22}$$

The solution of the least squares method is derived from the following equation by differentiating J by $\mathbf{q}$ as 0:

$$\begin{aligned} \frac{\partial J}{\partial \mathbf{q}} &= 2\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}_{p,k,l}{}^{t}\mathbf{F} \cdot {}_{p,k,l}\mathbf{F} \cdot \mathbf{q} \\ &\quad + 2\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}_{p,k,l}{}^{t}\mathbf{F} \cdot {}_{p,k}\mathbf{d} = \mathbf{0} \end{aligned} \tag{23}$$

From this Eq. (23), the solution of the least squares method is calculated with the following equation:

$$\hat{\mathbf{q}} = -\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}_{p,k,l}{}^{t}\mathbf{F}\cdot{}_{p,k,l}\mathbf{F}\right)^{-1}\cdot\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}_{p,k,l}{}^{t}\mathbf{F}\cdot{}_{p,k}\mathbf{d}\right) \quad (24)$$

This is the solution of the normal least squares method. In an actual situation, where various errors may arise, the airflow is often estimated to be negative. An optimum **q** with minimum J in the non-negative constraints can be obtained with the algorithm of the non-negative least squares method [15]. The basic equation is also the same in this case.

4. Equation residual analysis and estimated error matrix

One of the keys to evaluating the estimated errors is the residual of the equation. Not only measurement errors of concentration or gas generation quantities, but also various causes of error such as structural differences between equation models and actual phenomena are finally represented in the residual of the equation. Residual **v** is defined with the following equation:

$${}_{p,k,l}\mathbf{v} = {}_{p,k,l}\mathbf{F}\cdot\hat{\mathbf{q}} + {}_{p,k}\mathbf{d} \quad (25)$$

The matrix of the expected residual of the equation is defined with the following equation. Here, nt is total number of various conditions by p, k, and l:

$$E(\mathbf{e}\cdot{}^{t}\mathbf{e}) \cong \frac{1}{nt}\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}_{p,k,l}\mathbf{v}\cdot{}^{t}_{p,k,l}\mathbf{v} \quad (26)$$

Estimated error variance–covariance matrix of the airflow can be described with the following equation as the error propagation from this matrix of the expected equation residual:

$$\begin{aligned}\boldsymbol{\Lambda}_q = &\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}_{p,k,l}\mathbf{F}\cdot{}_{p,k,l}\mathbf{F}\right)^{-1}\cdot\\ &\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}_{p,k,l}\mathbf{F}\cdot E(\mathbf{e}\cdot{}^{t}\mathbf{e})\cdot{}_{p,k,l}\mathbf{F}\right)\cdot\\ &{}^{t}\left\{\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}_{p,k,l}\mathbf{F}\cdot{}_{p,k,l}\mathbf{F}\right)^{-1}\right\}\end{aligned} \quad (27)$$

The diagonal element is the error variance of the airflow, and the non-diagonal element is the covariance. The equation to calculate the coefficient of determinant is shown below. The residual sum of squares, $s(\mathbf{q})$,of all the conditions p, k, and l is defined with the Eq. (28):

$$s(\hat{\mathbf{q}}) = \sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}_{p,k,l}\mathbf{v}\cdot{}_{p,k,l}\mathbf{v} \quad (28)$$

The magnitude of this residual sum of squares is evaluated using the vector **y**, which will be defined by the next Eq. (29) and is called the total variation vector of gas generation. The gas generation rate is considered to vary depending on the term p and gas type k but is considered to have no relation with the gas absorption point l. However because of the definition of residual sum of squares by Eq. (28), to coincide the summing up number with this, and on the promise to apply the same **g** to the absorption point variation, **y** of the Eq. (29) is defined. Vector **d** was defined in Eq. (21):

$${}_{p,k,l}\mathbf{y} = {}_{p,k}\mathbf{d}. \quad (29)$$

Taking the difference from the term average of the gas generation rate vector **y**, the value taking the sum of squares under various conditions is expressed by Eq. (30) and called the total variation s_y:

$$\begin{aligned}s_y = &\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}({}_{p,k,l}\mathbf{y}-\bar{\mathbf{y}})\cdot({}_{p,k,l}\mathbf{y}-\bar{\mathbf{y}})\\ = &\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}_{p,k,l}\mathbf{y}\cdot{}_{p,k,l}\mathbf{y}\\ &-\frac{1}{nt}\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}^{t}_{p,k,l}\mathbf{y}\right)\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl} {}_{p,k,l}\mathbf{y}\right)\end{aligned} \quad (30)$$

The coefficient of determinant, COD, is calculated with Eq. (31). The maximum of this COD is unity, and a smaller COD indicates a larger estimation error:

$$\mathrm{COD} = 1 - \frac{s(\hat{\mathbf{q}})}{s_y} \quad (31)$$

5. Index of discrepancy from model premises

The mathematical model based on the tracer gas flow balance rests on several modeling premises. For example, the uniformity of concentration inside a zone and the invariability and linearity of parameters such as airflow rates or effective mixing volumes are assumed. In most actual measurement situations these premises are not adequately realized. Often this deviation from the model premises has a larger effect than the measurement error on the reliability of the ventilation estimation. A reliability evaluation index can be defined by taking the ratio of which numerator is the estimated standard deviation of the departure from model premises and the denominator is the error standard deviation derived solely from measurement error. The discrepancy from the model premises appears in the equation residual described in the previous section. Therefore, initially only the error propagation to estimated parameters from the measurement errors for the gas generation rate and gas concentration is described. That is, an equation for the measurement error variance–covariance matrix is deduced corresponding to the combined equation system (14) and (20).

Measured values are considered to be true values with the measurement error **s** added. Here the vectors containing measurement values, measurement errors and error standard deviations are defined as follows.

The vector of concentration measurement will now be defined. When only the measurement error is considered, concentration uniformity is satisfactory and does not vary between sampling points inside a zone. Therefore the subscript l is eliminated:

$${}_{p,k}\mathbf{c} = {}^{t}({}_{p,k}\bar{c}_1,\ldots,{}_{p,k}\bar{c}_n,{}_{p,k}\bar{c}_{n+1}) \quad (32)$$

The vector of measurement errors is defined as follows:

$${}_{p,k}\mathbf{s}_c = {}^{t}({}_{p,k}s_{c1},\ldots,{}_{p,k}s_{cn},{}_{p,k}s_{cn+1}) \quad (33)$$

The true value of a measurement is denoted with a double bar superscript. Therefore, a measured vector ${}_{p,k}\mathbf{c}$ is regarded as the sum of the true value vector and the error vector ${}_{p,k}\mathbf{s}_c$:

$$\begin{aligned}{}_{p,k}\mathbf{c} = {}_{p,k}\bar{\bar{\mathbf{c}}} + {}_{p,k}\mathbf{s}_c = {}^{t}(&{}_{p,k}\bar{\bar{c}}_1 + {}_{p,k}s_{c1},\ldots,{}_{p,k}\bar{\bar{c}}_n + {}_{p,k}s_{cn},\\ &{}_{p,k}\bar{\bar{c}}_{n+1} + {}_{p,k}s_{cn+1})\end{aligned} \quad (34)$$

The measurement error standard deviation vector is defined as follows, where the variance is defined as the square of the standard deviation:

$${}_{p,k}\boldsymbol{\sigma}_c = {}^{t}({}_{p,k}\sigma_{c1},\ldots,{}_{p,k}\sigma_{cn},{}_{p,k}\sigma_{cn+1}) \quad (35)$$

The vectors for the gas generation rate are defined similarly:

$$_{p,k}\mathbf{g} = {}^{t}({}_{p,k}\bar{\bar{g}}_1, \ldots, {}_{p,k}\bar{\bar{g}}_n) \quad (36)$$

$$_{p,k}\mathbf{s}_g = {}^{t}({}_{p,k}s_{g1}, \ldots, {}_{p,k}s_{gn}) \quad (37)$$

$$_{p,k}\mathbf{g} = {}_{p,k}\bar{\bar{\mathbf{g}}} + {}_{p,k}\mathbf{s}_g = {}^{t}({}_{p,k}\bar{\bar{g}}_1 + {}_{p,k}s_{g1}, \ldots, {}_{p,k}\bar{\bar{g}}_n + {}_{p,k}s_{gn}) \quad (38)$$

$$_{p,k}\boldsymbol{\sigma}_g = {}^{t}({}_{p,k}\sigma_{g1}, \ldots, {}_{p,k}\sigma_{gn}) \quad (39)$$

Combining the tracer gas flow balance simultaneous equation system (14) and the airflow continuity equation system (20), the measurement error vector equation is defined as follows.

Considering that the use of true values makes the equation errors 0, the following equation can be written:

$$_{p,k}\boldsymbol{\varepsilon} = \begin{bmatrix} \mathbf{Q}\cdot{}_{p,k}\mathbf{c} + {}_{p,k}\mathbf{g} \\ \mathbf{C}\cdot\hat{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}\cdot({}_{p,k}\bar{\bar{\mathbf{c}}} + {}_{p,k}\mathbf{s}_c) + ({}_{p,k}\bar{\bar{\mathbf{g}}} + {}_{p,k}\mathbf{s}_g) \\ \mathbf{C}\cdot\hat{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}\cdot{}_{p,k}\mathbf{s}_c + {}_{p,k}\mathbf{s}_g \\ \mathbf{C}\cdot\hat{\mathbf{q}} \end{bmatrix} \quad (40)$$

The lower half of the vector represents constraints on the airflow balance and can be considered to be **0** as it has no relation to measurement errors:

$$_{p,k}\boldsymbol{\varepsilon} = \begin{bmatrix} \mathbf{Q}\cdot{}_{p,k}\mathbf{s}_c + {}_{p,k}\mathbf{s}_g \\ \mathbf{C}\cdot\hat{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}\cdot{}_{p,k}\mathbf{s}_c + {}_{p,k}\mathbf{s}_g \\ \mathbf{0} \end{bmatrix} \quad (41)$$

To calculate the expected matrix by this error vector, by observing Eq. (41), the upper half of this Eq. (41) from Eq. (14) must be defined as next Eq. (42):

$$_{p,k}\boldsymbol{\varepsilon}_u = \mathbf{Q}\cdot{}_{p,k}\mathbf{c} + {}_{p,k}\mathbf{g} = \mathbf{Q}\cdot({}_{p,k}\bar{\bar{\mathbf{c}}} + {}_{p,k}\mathbf{s}_c) + ({}_{p,k}\bar{\bar{\mathbf{g}}} + {}_{p,k}\mathbf{s}_g) = \mathbf{Q}\cdot{}_{p,k}\mathbf{s}_c + {}_{p,k}\mathbf{s}_g \quad (42)$$

The expected error matrix of Eq. (42) (due to measurement errors) can be calculated with the standard deviation σ_c and σ_g based on the characteristics of the instruments with Eq. (43). Here, such features were utilized that the covariance between the errors $_{p,k}s_c$ and $_{p,k}s_g$ is 0 and the covariance between elements in these two vectors is also 0. The notation "diag" indicates a matrix composed of only the diagonal elements in the matrix in these parentheses:

$$\begin{aligned} _{p,k}E({}_{p,k}\boldsymbol{\varepsilon}_u\cdot{}^{t}_{p,k}\boldsymbol{\varepsilon}_u) &= \mathbf{Q}\cdot{}_{p,k}E({}_{p,k}\mathbf{s}_c\cdot{}^{t}_{p,k}\mathbf{s}_c)\cdot{}^{t}\mathbf{Q} + {}_{p,k}E({}_{p,k}\mathbf{s}_g\cdot{}^{t}_{p,k}\mathbf{s}_g) \\ &= \mathbf{Q}\cdot\mathrm{diag}({}_{p,k}\boldsymbol{\sigma}_c\cdot{}^{t}_{p,k}\boldsymbol{\sigma}_c)\cdot{}^{t}\mathbf{Q} + \mathrm{diag}({}_{p,k}\boldsymbol{\sigma}_g\cdot{}^{t}_{p,k}\boldsymbol{\sigma}_g) \end{aligned} \quad (43)$$

Accordingly, the expected matrix of $_{p,k}\varepsilon$ added to the vector of the lower half can be calculated with Eq. (44):

$$_{p,k}E({}_{p,k}\boldsymbol{\varepsilon}\cdot{}^{t}_{p,k}\boldsymbol{\varepsilon}) = \begin{bmatrix} {}_{p,k}E({}_{p,k}\boldsymbol{\varepsilon}_u\cdot{}^{t}_{p,k}\boldsymbol{\varepsilon}_u) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (44)$$

The estimated error variance–covariance matrix of the airflow rate from measurement error is calculated with Eq. (45):

$$_{m}\boldsymbol{\Lambda}_q = \left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl}{}^{t}_{p,k,l}\mathbf{F}\cdot{}_{p,k,l}\mathbf{F}\right)^{-1}\cdot\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl}{}^{t}_{p,k,l}\mathbf{F}\cdot{}_{p,k}E({}_{p,k}\boldsymbol{\varepsilon}\cdot{}^{t}_{p,k}\boldsymbol{\varepsilon})\cdot{}_{p,k,l}\mathbf{F}\right)\cdot{}^{t}\left\{\left(\sum_{p=1}^{np}\sum_{k=1}^{nk}\sum_{l=1}^{nl}{}^{t}_{p,k,l}\mathbf{F}\cdot{}_{p,k,l}\mathbf{F}\right)^{-1}\right\} \quad (45)$$

The degree to which the premises of the mathematical model are satisfied by the actual phenomena can be seen by comparing the diagonal element size of $_{m}\boldsymbol{\Lambda}_q$ with that of $\boldsymbol{\Lambda}_q$. Here, the jth diagonal element of $_{m}\boldsymbol{\Lambda}_q$ is expressed by $_{m}\sigma^2_{\lambda jj}$, and the jth diagonal element of $\boldsymbol{\Lambda}_q$ by $\sigma^2_{\lambda jj}$. Taking the square root of these diagonal elements, the discrepancy ratio β of the model premises of the following equation and its average value are defined:

$$\beta_j = \frac{\sigma_{\lambda j,j}}{_{m}\sigma_{\lambda j,j}} \quad (46)$$

$$\bar{\beta} = \frac{1}{nq}\sum_{j=1}^{nq}\beta_j \quad (47)$$

When the magnitude of β is relatively large, there may be a problem with the measurement conditions or the model. In this case it is better to redo the measurement or to remake the model.

6. Irrationality index of estimated negative airflow

The results of the non-negative least squares method and the normal least squares method agree if the actual conditions and model premises are highly compatible and there are no measurement errors. In reality, though, differences appear in the estimated results. An index to evaluate the appropriateness of measurements will now be determined by making use of these differences. In the [i] result of estimated airflow of the normal least squares method, [ii] result assuming the negative value of result of [i] as airflow in reverse direction, and [iii] result of estimated airflow by the non-negative least squares method, describing the mth estimated result of the three types of estimation methods as $_{m}q_{ij}$, the average value of the three types is defined by Eq. (48). Variation among the three types is calculated with the variance σ^2_{qij} of Eq. (49) based on the average of the estimated results of the three types for individual airflows:

$$\bar{q}_{i,j} = \frac{1}{3}\sum_{m=1}^{3}||{}_{m}q_{i,j}|| \quad (48)$$

$$\sigma^2_{qij} = \frac{1}{3}\sum_{m=1}^{3}(||{}_{m}q_{i,j}|| - \bar{q}_{i,j})^2 \quad (49)$$

The average value σ^2_q of all airflow rates is obtained from Eq. (50). The average of all the estimated airflow rates is calculated with the following Eq. (51) from the solution by the non-negative least squares method:

$$\bar{\sigma}^2_q = \frac{1}{nq}\sum_{i=1}^{n+1}\sum_{j=1}^{n+1}\sigma^2_{qij} \quad (50)$$

$$\bar{q} = \frac{1}{nq}\sum_{i=1}^{n+1}\sum_{j=1}^{n+1}||{}_{3}q_{i,j}|| \quad (51)$$

A negative flow rate irrationality index R_a is defined by the following Eq. (52). This index will also be useful to judge the appropriateness of the measurement:

$$R_a = \frac{\bar{\sigma}_q}{\bar{q}} \quad (52)$$

7. Calculation examples

A calculation program (PFTSID) was developed based on the above-described theory and simulated measured values were prepared and verified through computer simulation. A four-zone model was assumed, and to calculate the ventilation rates, gas concentrations and changes of ventilation flow rate and gas concentration were simulated for one week. Four samplers were provided in each zone of the model (16 in all). Average concentrations for a week were added to the error using random number generation and deemed to be the measured values of the average concentration for the period. Using these simulated measured values, airflows were estimated and the error was evaluated. This theory can consider variation in multiple periods, but in this example just one period, $p = 1$, was used.

The thermal and airflow network model calculation program NETS [16] was used for modeling and simulation of the ventilation and gas concentration variation calculation. Fig. 1 is a plan view of each floor of the ventilation calculation model. This model only considers natural ventilation caused by infiltration. The driving force of ventilation is wind pressure and stack effects. For the outside air temperature and the direction and velocity of the wind, annual standard meteorological data for every hour in Tokyo was used. The simulation time interval was 15 min. NETS generated and applied the outside air temperature and direction and velocity of wind for 1 h at 15 min intervals by linear interpolation. It was assumed that there are cracks equivalent to openings of 5 cm^2/m^2 in the outer wall, 2 cm^2/m^2 in the roof and floor, and 15 cm^2/m^2 in the partition wall and the floor on the second floor. The capacities of zones 1 through 4 were 100, 25, 62.5 and 62.5 m^3, respectively.

Heights of the floor on the first and second floors, and the roof from the ground surface were 0, 2.5 and 5.0 m, respectively. Two airflow paths of different heights were provided by dividing the height of each wall into two parts to express the airflow in both directions caused by the temperature difference between zones with a wall interposed. The airflow paths were provided for each orientation of the wall for the purpose of considering the difference in the effect of wind pressure due to wind direction. The pressure difference between the space above and under the floor is uniform along the floor surface in the model, and one airflow path is enough.

It was assumed that the room temperature changes in each zone in a day have a pattern as shown in Fig. 2. The wind pressure coefficient is positive when the outer wall faces windward and negative when facing leeward as shown in Fig. 3. The building stands in a congested area and experiences only a small amount of wind pressure.

Fig. 4 shows the variation of the difference between the average temperature in all zones and the outside air temperature. Fig. 5 shows changes in the ventilation rate of the whole building. Figs. 6–9 show the changes of concentration of four types of gases in each zone. Particularly zone 2 is often on the windward side, and much outside air enters due to the stack effect, making it difficult for gas to enter from other zones.

By averaging the changes of airflow in 7 days by periods, the airflow distribution shown in Fig. 10 was obtained. Such airflows shall be the correct answer to be estimated by this theory.

By adding errors based on the random number generation to the true values as shown in Table 1, assuming the standard deviation of measurement error of the gas generation rate to be 1 and that of the gas concentration to be 0.001, simulated measurement values were generated. The gas concentration was measured at 4 points in each zone. The symbol g represents the gas generation rate and c the gas concentration, and the left- and right-hand subscript numbers are defined in the same way as described above. The right-hand subscript numbers represent zone numbers. The left-hand subscript numbers represent the term number, gas type number, and point number in the zone to the left, including commas.

The conventional deterministic method yields 20 airflows, including ones which do not exist. The least squares method makes calculations for 16- and 14-airflow models (depending on the existence of airflow between zones 2 and 4). The airflows were estimated by three methods: a method using negative airflows as they are, according to the normal least squares method, a method assuming negative airflows to be in the reverse direction, and the

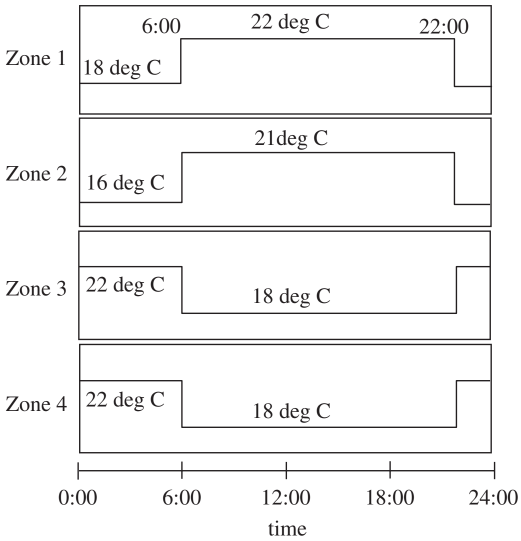

Fig. 2. Zone air temperature schedule.

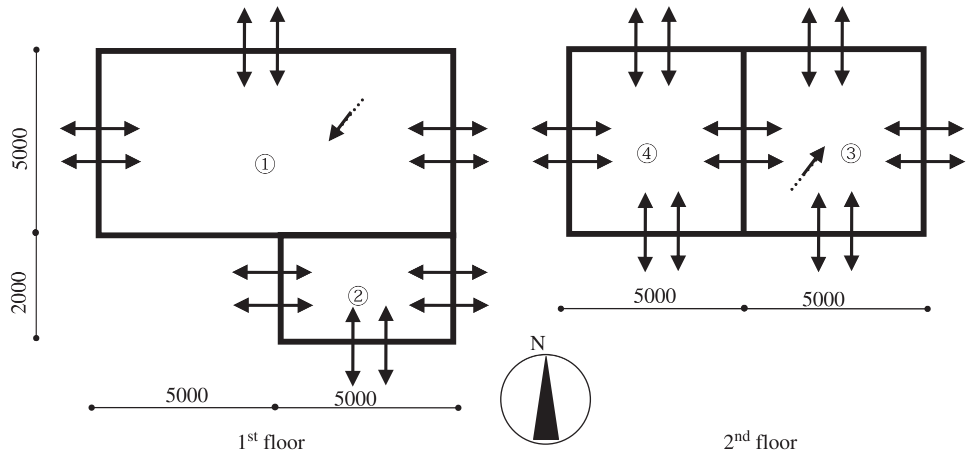

Fig. 1. Four-zone calculation experimental model.

non-negative least squares method. Comparison of these results with true values is shown in Table 2.

Various indices for evaluating errors are shown. Table 3 shows the calculation of the discrepancy ratio β with respect to premises of models based on the results of the non-negative least squares method with 14 airflows. Table 4 shows the average of the irrationality index of estimated negative airflows and β. The coefficient of determinant was also calculated, yielding a value close to 1 in any model.

8. Discussion and future tasks

Table 2 shows four negative airflows produced by the conventional method, displaying a relatively large difference from the non-negative least squares method. Though the estimated accuracy depending on the number of the estimated airflows by the non-negative least squares method is a little inferior in the 20 airflow estimation compared to that of other methods, the difference between 16 and 14 airflows is relatively small. In the case of 16 airflows, one airflow is 0.02 m^3/h and the corresponding opposite-direction flow is 0.03 m^3/h, both of which should be 0.

By the present method, the necessary and sufficient number of flow rates are estimated which are supposed to exist from the geometrical relation between zones. The estimation with non-existence flow rates makes the estimation precision worse and often produces negative flow rates. The negative flow rates are against the law of physics. The negative flow rates are the results for meeting the requirement of simultaneously satisfying both flow balances of air and gas. Therefore, if the negative flow rate is regarded as the opposite direction one, then the gas mass flow balance will not be satisfied even though the air flow balance is satisfied. The multi-zonal model of gas diffusion system in buildings is mathematically similar to the multi-nodal model by the spatial discretization of the heat transfer system. The flow rate is corresponding to thermal conductance. Negative conductance in the heat transfer system causes irrational heat flow from the lower temperature node to the higher temperature node and is against the law of thermodynamics. As long as there is any error cause of not only measurement but also discrepancy of model premise from real phenomenon, gas and air flow balance equations will not be perfectly satisfied. Because of the restriction of PFT method, the basic equation model has to suppose the steady-state assumption. The discrepancy from this premise and real phenomenon cannot be ignored. Consequently, negative flow rates may be produced. The flow rates estimation method should be considered an optimization method to minimize the equation error with the non-negative flow rate constraints.

As described in the section Basic Mathematical Model, this method estimates the concentration-weighted term-average airflow rates and not simply the term-average airflow rates. This is verified in Table 3. The sum of estimated airflow rates is 421.02 m^3/h, slightly smaller than the real value of 425.2 m^3/h.

The discrepancy ratios for the model premises are shown in Table 3. In this example, the assumption of constant ventilation airflow, which is one of the model assumptions, is not valid and its variation produces effects in addition to the effects of measurement errors. The average of β is 0.19. In this context, a case considering only the effect of measurement errors under constant

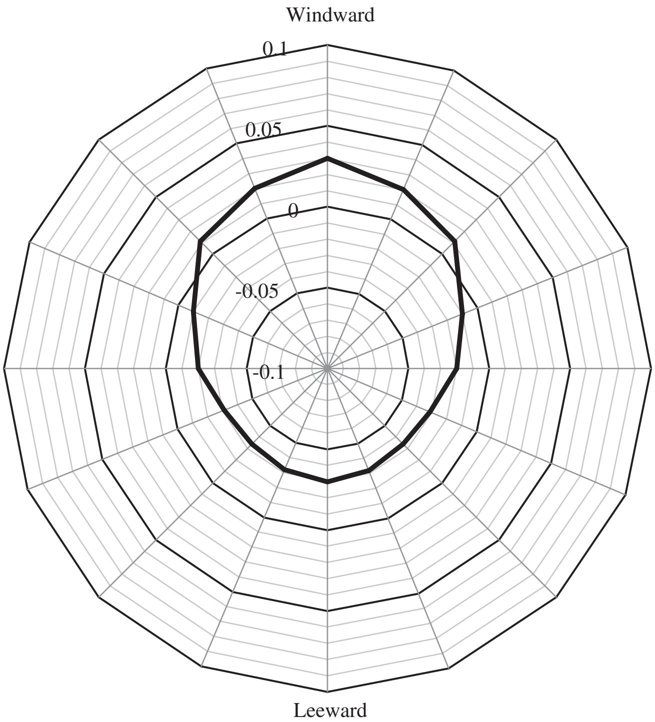

Fig. 3. Wind pressure coefficient model.

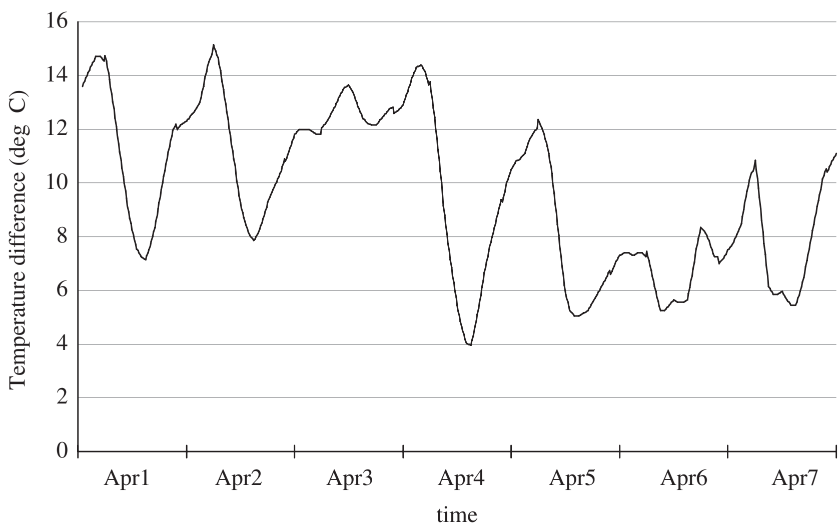

Fig. 4. Average air temperature difference inside and outside the building.

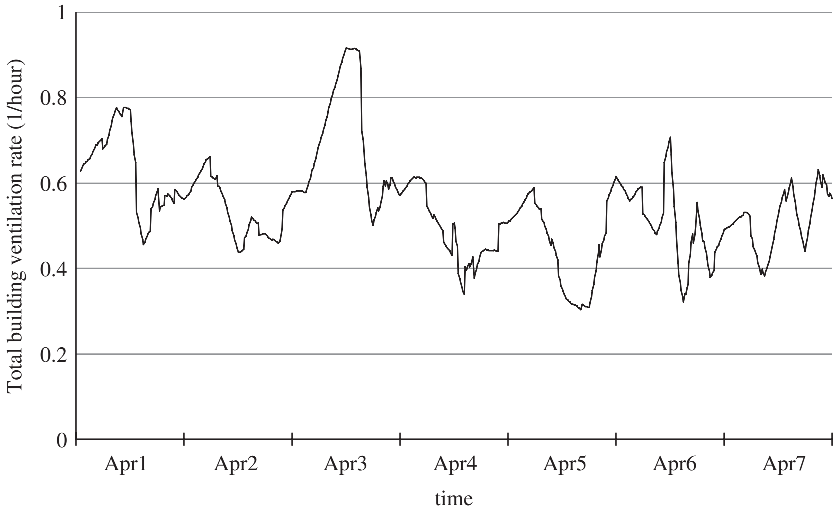

Fig. 5. Simulated total building ventilation rate.

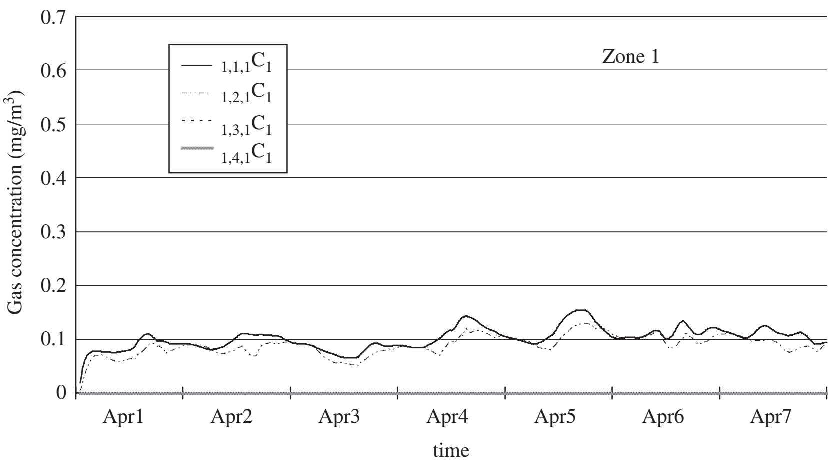

Fig. 6. Tracer gas concentrations of each gas in zone 1.

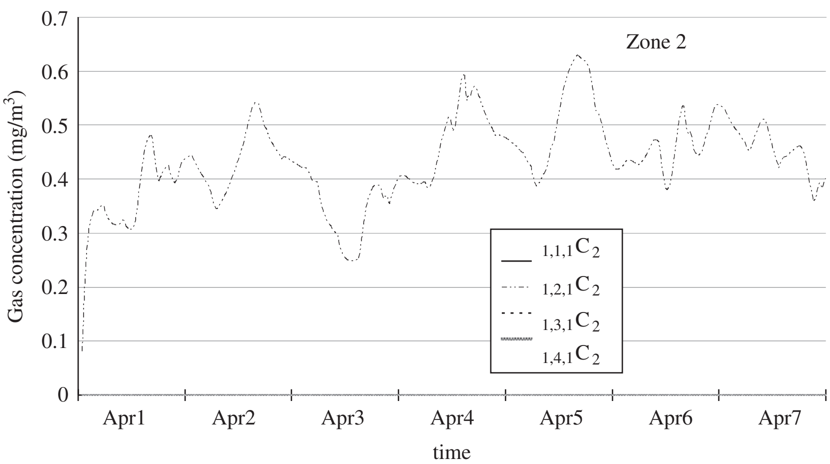

Fig. 7. Tracer gas concentrations of each gas in zone 2.

airflow was calculated, yielding an average β as small as 0.06. The index of discrepancy from model premises seems to show appropriate differences. β seems to vary over a wide range according to the case.

The irrationality index of estimated negative airflow as shown in Table 4 is 0.090 for the conventional method where four negative airflows were generated, but is as small as 0.078 in the estimation of 14 airflows without negative values. With the

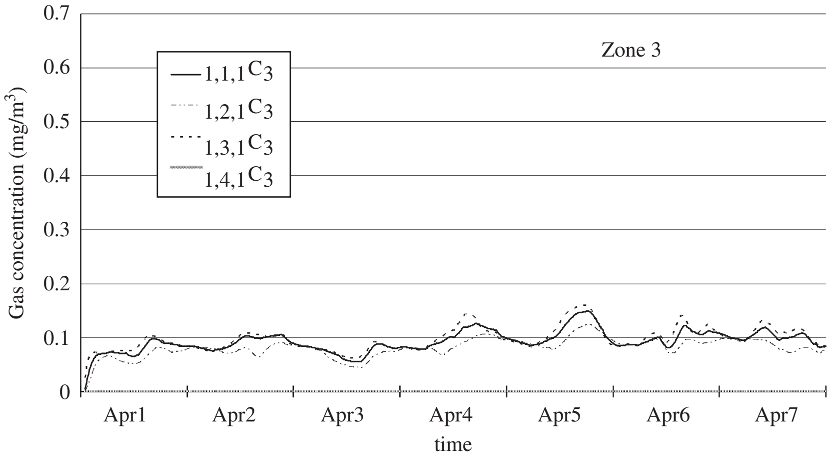

Fig. 8. Tracer gas concentrations of each gas in zone 3.

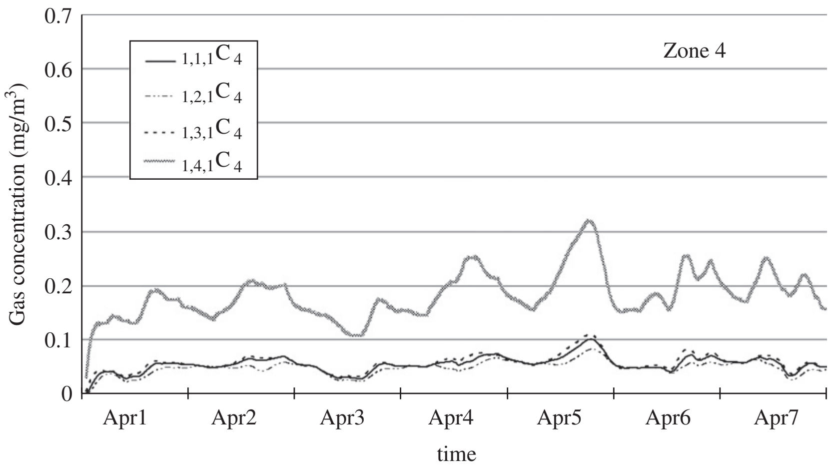

Fig. 9. Tracer gas concentrations of each gas in zone 4.

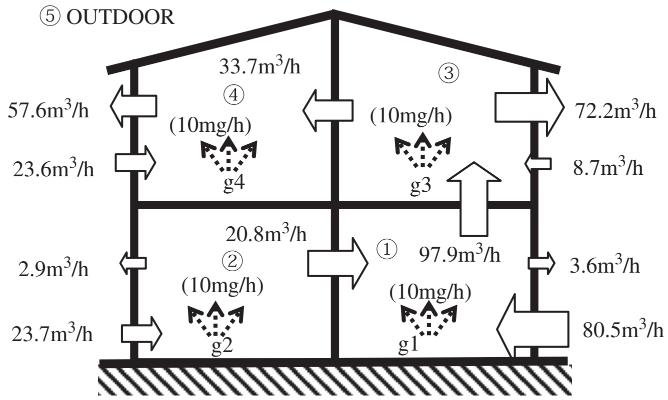

Fig. 10. One week averaged airflow rates and gas generation rates.

Table 1
True values of gas generation rate and gas concentration

Gas generation rate (mg/h)			
$_{1,1}g_1$	$_{1,2}g_2$	$_{1,3}g_3$	$_{1,4}g_4$
10	10	10	10
Gas concentration (mg/m^3)			
$_{1,1,1}C_1$	$_{1,2,1}C_1$	$_{1,3,1}C_1$	$_{1,4,1}C_1$
0.10167	0.08822	0	0
$_{1,1,1}C_2$	$_{1,2,1}C_2$	$_{1,3,1}C_2$	$_{1,4,1}C_2$
0	0.43329	0	0
$_{1,1,1}C_3$	$_{1,2,1}C_3$	$_{1,3,1}C_3$	$_{1,4,1}C_3$
0.09322	0.08077	0.09755	0
$_{1,1,1}C_4$	$_{1,2,1}C_4$	$_{1,3,1}C_4$	$_{1,4,1}C_4$
0.05501	0.04777	0.05751	0.18227

conventional method, the index of discrepancy from model premises is as large as 0.27.

It was also recognized that error evaluation cannot be achieved solely by the coefficient of determinant. The error evaluation indices β and R_a will be useful in devising better model structures. However, many cases should be studied in the future to set the criterion for a judgment value in advance.

In this example, the effects of multiple terms, more gas types than zones, and uneven gas concentration within a zone, which are features of this theory and can be considered, have not yet been studied for their effect on the evaluation of reliability. Only the influence of changes of ventilation with time was investigated. Non-uniform concentration at 4 points in a single zone in this case was due solely to variations within the normal probability distribution of measurement error. However, as the basic mathematical model has omitted the term for changes with time, proper residual analysis and error evaluation might be

Table 2
Results of various estimation methods (m^3/h)

Leeward *i* zone	Windward *j* zone	Conventional method	Non-negative LS, *q* 20 assumed	Non-negative LS, *q* 16 assumed	Non-negative LS, *q* 14 assumed	Real *q* value
5	1	−10.71	0	0	0	3.6
5	2	3.02	2.02	2.69	2.72	2.9
5	3	82.21	75.77	72.73	72.76	72.2
5	4	55.41	55.08	56.17	56.07	57.6
1	5	75.05	75.85	75.75	75.75	80.5
2	5	22.29	22.14	22.32	22.34	23.7
3	5	9.27	11.51	10.68	10.65	8.7
4	5	23.33	23.38	22.84	22.81	23.6
1	2	19.34	19.64	19.63	19.63	20.8
2	1	−0.24	0	0	0	0
3	4	1.05	1.05	0.67	0.73	0
4	3	32.70	32.77	33.99	33.99	33.7
1	3	0.17	2.86	4.10	4.10	0
3	1	105.18	98.35	99.47	99.47	97.9
2	4	0.09	0.22	0.02	–	0
4	2	0.17	0.20	0.03	–	0
4	1	0.29	0	–	–	0
1	4	−0.05	0	–	–	0
3	2	−0.20	0.49	–	–	0
2	3	0.20	0	–	–	0

qx: *x* is total number of flow rates estimated.

Table 3
Discrepancy ratio of model premises (m^3/h)

Leeward *i* zone	Windward *j* zone	Estimated *q* value	Real *q* value	δ error	σ_λ	$m\sigma_\lambda$	β
5	1	0	3.6	−3.60	2.80	10.38	0.27
5	2	2.72	2.9	−0.18	0.31	2.18	0.14
5	3	72.76	72.2	0.56	2.49	8.35	0.30
5	4	56.07	57.6	−1.53	0.85	3.90	0.22
1	5	75.75	80.5	−4.75	0.87	4.12	0.21
2	5	22.34	23.7	−1.36	0.08	4.07	0.02
3	5	10.65	8.7	1.95	1.06	3.93	0.27
4	5	22.81	23.6	−0.79	0.32	3.38	0.10
1	2	19.63	20.8	−1.17	0.32	1.54	0.21
2	1	0	0	0.00	0.09	4.91	0.02
3	4	0.73	0	0.73	0.75	2.77	0.27
4	3	33.99	33.7	0.29	0.34	3.59	0.10
1	3	4.10	0	4.10	1.10	5.21	0.21
3	1	99.47	97.9	1.57	1.64	6.07	0.27
Average value	–	–	–	–	0.93	4.60	0.19

Table 4
Irrationality index of estimated negative airflow and average of β

Estimated-q 14		
$\bar{\sigma}_q^2$	$\bar{\sigma}_q$	$\bar{\beta}$
5.55	2.35	0.19
$\bar{q}$	$R_a = \bar{\sigma}_q/\bar{q}$	
30.07	0.078	
Conventional method		
$\bar{\sigma}_q^2$	$\bar{\sigma}_q$	$\bar{\beta}$
3.60	1.90	0.27
$\bar{q}$	$R_a = \bar{\sigma}_q/\bar{q}$	
21.07	0.090	

qx: *x* is total number of flow rates estimated.

limited. This is one reason why the coefficient of determinant was usually 1.

The present theory requires that the number of sampling points in all zones be the same. However, the actual number of sampling points in each zone will vary according to zone size. It is natural to use fewer sampling points in a smaller zone than in a larger one. For this situation the maximum number of sampling points in all the zones is taken to be the ‘‘sampling points number *nl*’’ and in zones with fewer, the measured data can be employed repeatedly to match the maximum sampling points. This is reasonable because the concentration uniformity is higher in a smaller zone than in a larger one.

This measurement method is essentially, however, not a measuring method to follow up the changes with time. Even if the ventilation rate may change a little when a door or window is opened or closed, it is likely that a concentration-weighted term average airflow rate that is close to the simple term average may be estimated with adequate accuracy over a long period. Therefore, error evaluation theory as described in this paper is also important from this viewpoint.

9. Conclusions

Statistical airflow estimation methods were devised for multi-zonal buildings based on the non-negative least squares method and the reliability evaluation method and were compared with the conventional deterministic method. The estimated flow rates are defined in this theory as the concentration-weighted term average flow rates and not the simple term average flow rates. The index β of discrepancy from model premises and the irrationality index R_a of the estimated negative airflow were introduced in addition to the conventional statistical index. This theory allows more accurate estimation of airflow than the conventional method. With the conventional method, only the effects of measurement errors can be evaluated. The proposed evaluation indices β and R_a in this theory make it possible to evaluate uncertainty due to nonconformity between the assumptions of an estimation method and the actual conditions. A calculation program (tentatively named PFTSID) based on this theory was developed, and part of the theory was verified with a numerical experiment. Limits due to the basic mathematical assumption of a steady state were discussed, but the significance of the method for long-term simple measurement was described.

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